1.5 Laws of Proposition logic

$\boxed{p \to q \equiv \neg p \vee q}$

- This rule can also apply to compound propositions.

$$(\neg t \wedge r) \to (\neg s \vee t) \equiv \neg(\neg t \wedge r) \vee (\neg s \vee t)$$

Laws of propositional Logic

Idempotent Laws	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative Laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative Laws	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive Laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity Laws	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination Laws	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double Negation Law	$\neg\neg p \equiv p$	
Complement laws	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's Laws	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption Laws	$p \vee (q \wedge p) \equiv p$	$p \wedge (q \vee p) \equiv p$
Conditional Identities	$p \to q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

1.6 Predicates and Qualifiers

"x is an odd number" $\neq$ proposition

- because x is not specified

<u>Predicate</u>: A logical statement with a truth value that is a function of 1 or more variables.

The <u>Domain</u> of a variable in a predicate is the set of all possible values for the variable.

"The city has a population over 1,000,000" = predicate

- because it has a variable
- the variable is "city"

- "$\forall x\, P(x)$" is read "for all x, $P(x)$"
  - This statement asserts that $P(x)$ is true for every value of x in domain.

- The symbol $\forall$ is a <u>universal quantifier</u>
- The statement "$\forall x\, P(x)$" is a <u>universally quantified statement.</u>

- "$\forall x\, P(x)$" = proposition
  - because it is either T or F

- <u>Arbitrary element</u>: nothing is assumed about the element other than the fact that it is in the domain.

- <u>Counterexample</u> for a universally quantified statement is an element in the domain for which the predicate is false

- If the domain for variable x is empty, then the statement $\forall x\, P(x)$ is True because there are no elements in the domain for which P(x) is false.

- "$\exists x\, P(x)$" is read "there exists an x such that P(x)"
  - This statement asserts that $P(x) = T$ for at least one x in domain

- The symbol $\exists$ is an existential qualifier.
- The statement "$\exists x\, P(x)$ is called an existentially quantified statement.
  - It is a proposition because it is either true or false

- An example for an existentially qualified statement is an element in the domain for which the predicate is true.

1.7 Quantified Statements

- The universal and existential quantifiers are generally called quantifiers.

Quantified Statement: A logical statement that includes a universal or existential quantifier.

- A variable x in the predicate P(x) is called a free variable.
- The variable x in $\forall x\, P(x)$ is a bound variable because the variable is bound to a quantifier.

1.8 De Morgan's Law for quantified statements

$$\boxed{\neg \forall x\, F(x) \equiv \exists x \neg F(x)}$$

$$\neg (F(a_1) \wedge F(a_2) \ldots \wedge F(a_n)) \equiv \neg F(a_1) \vee \neg F(a_2) \ldots \vee \neg F(a_n)$$

$$\boxed{\neg \exists x\, P(x) \equiv \forall x \neg P(x)}$$

$$\neg (P(a_1) \vee P(a_2) \vee \ldots \vee P(a_n)) \equiv \neg P(a_1) \wedge \neg P(a_2) \ldots \wedge \neg P(a_n)$$